11.1: 31-36

31. (a) C (b) II

Reasons: This function is periodic in both x and y, and the function is the same when x is interchanged with y, so its graph is symmetric about the plane $y = x$. In addition, the function is 0 along the x- and y-axes. These conditions are satisfied only by C and II.

32. (a) A (b) IV

Reasons: This function is periodic in y but not x, a condition satisfied only by A and IV. Also, note that traces in $x = k$ are cosine curves with amplitude that increases as x increases.

33. (a) F (b) I

Reasons: This function is periodic in both x and y but is constant along the lines $y = x + k$, a condition satisfied only by F and I.

34. (a) E (b) III

Reasons: This function is periodic in both x and y, but unlike the function in Exercise 33, it is not constant along lines such as $y = x + \pi$, so the contour map is III. Also notice that traces in $y = k$ are vertically shifted copies of the sine wave $z = \sin x$, so the graph must be E.

35. (a) B (b) VI

Reasons: This function is 0 along the lines $x = \pm 1$ and $y = \pm 1$. The only contour map in which this could occur is VI. Also note that the trace in the xz-plane is the parabola $z = 1 - x^2$ and the trace in the yz-plane is the parabola $z = 1 - y^2$, so the graph is B.

36. (a) D (b) V

Reasons: This function is the same if x is interchanged with y, so its graph is symmetric about the plane $y = x$. Several of the contour maps exhibit this symmetry, but the function is not periodic in x or y, so the correct contour map is V. Also, the values of z approach 0 as we use points farther from the origin. The only graph that shows this behavior is D.

11.2: 10, 32, 34; Plus one more

10. $f(x, y) = 6x^3y/(2x^4 + y^4)$. On the x-axis, $f(x, 0) = 0$ for $x \neq 0$, so $f(x, y) \to 0$ as $(x, y) \to (0, 0)$ along the x-axis. Approaching $(0, 0)$ along the line $y = x$ gives $f(x, x) = 6x^4/(3x^4) = 2$ for $x \neq 0$, so along this line $f(x, y) \to 2$ as $(x, y) \to (0, 0)$. Thus the limit does not exist.

32. $f(x,y) = \begin{cases} \dfrac{xy}{x^2 + xy + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ The first piece of f is a rational function defined everywhere except at the origin, so f is continuous on $\mathbb{R}^2$ except possibly at the origin. $f(x, 0) = 0/x^2 = 0$ for $x \neq 0$, so $f(x, y) \to 0$ as $(x, y) \to (0, 0)$ along the x-axis. But $f(x, x) = x^2/(3x^2) = \frac{1}{3}$ for $x \neq 0$, so $f(x, y) \to \frac{1}{3}$ as $(x, y) \to (0, 0)$ along the line $y = x$. Thus $\lim_{(x,y)\to(0,0)} f(x, y)$ doesn't exist, so f is not continuous at $(0, 0)$ and the largest set on which f is continuous is $\{(x, y) \mid (x, y) \neq (0, 0)\}$.

34. $$\lim_{(x,y)\to(0,0)} (x^2 + y^2)\ln(x^2 + y^2) = \lim_{r\to 0^+} r^2 \ln r^2 = \lim_{r\to 0^+} \frac{\ln r^2}{1/r^2}$$

$$= \lim_{r\to 0^+} \frac{(1/r^2)(2r)}{-2/r^3} \quad \text{[using l'Hospital's Rule]} \quad = \lim_{r\to 0^+} (-r^2) = 0$$